	ESC	OLA	UNIVERSITÀRIA D'ENGINY	ERIA TÈCI	NICA INDUSTRIA	L DE BARCELO	ONA		
			Degree in EN	NGINEERIN	G (All courses)				
	Tècnica Indu	i <mark>strial d</mark> pla Indus	d'Enginyeria 2 Barcelona trial de Barcelona CNICA DE CATALUNYA	of cour	se (English)	)	**★ ★ E C T S ★		
						Acronym:	CMA		
Subject:		CC	MPUTATIONAL MECHA	Code:					
				Version:					
Type:			Total Credits ECTS:	б	Total hours/week:		4		
Language:			In Classroom Credits (Theory):		In classroom hours/		2		
Semester:			In Classroom Credits (Problems) Laboratory Credits:	):	In classroom hours/ Laboratory hours/v		0 2		
Level:			Out of the classroom Credits:		Out of the classroor		Z		
Descriptors									
(BOE):	5								
Coordinato			Di Capua						
Teaching st		niel L	Di Capua, José María Carbonell.						
Prerequisite									
Co-requisite General			e is particularly addressed to the	• • • • • • • • • • • • • • • • • • • •	. 1		P.1		
Objectives:	there const many The t	fore ructic cothe collow 1. In 2. A 3. A F 4. A 5. A th 5. K 7. C	I here in a broad sense. The Fi applicable to the analysis of st ons, mechanical components and a r applications. Ving general objectives of this count outroduction to the basic concepts of cquisition of a specific vocabular bility to read, correctly interpret a EM. bility to handle basic FEM softwa cquire basic knowledge of litera the FEM. Inowledge of sources of informati apacity for independent learning in conis 1: Know the first concepts of	ructures in o structural par irse can be co of the resolut: y of FEM. and understan are. ture and abil on, institutio issues within	civil engineering co ts in automotive, na onsidered: ion problems of soli and texts, figures and ity to perform litera nal and private, rela the scope of the FE	onstructions, build aval and aerospace of d mechanics with t tables in technical ature searches relat ted to the FEM.	ings and historical engineering, among he FEM. literature related to ting to the scope of		
Specific obj by topic:	ectives	T T	<ul> <li>opic 1: Know the first concepts o analysis of bar structures. I matrix structural analysis. structure in different eleme global equilibrium equation</li> <li>opic 2: Know and be able to und bars using one-dimensiona the FEM, such as discret stiffness matrix and the equilibrium equation</li> <li>opic 3: Know and be able to ap convergence of the FE solut</li> <li>opic 4: Know and be able to the Know and be able to the</li> </ul>	Be able to un Know and ents, the equi ns of the stru- lerstand the F al (1D) bar el tization, inter uivalent noda oply other ge tion, the type e basics conc	derstand the relation be able to understa librium of the indivi- cture from the contre FEM formulation fo ements. Know and f rpolation, shape fu al force vector for the neral concepts such s of errors, numerica cepts of structures to	n betwen FEM and and clearly the con- idual elements and ibutions of the diffe- r the analysis of sin- be able to apply the nctions, numerical e element. n as the patch test, al integration.	the methodology of ncept of splitting a the assembly of the erent elements. mple axially loaded e key ingredients of integration of the , the conditions for on of 2D elasticity.		
			Know and be able to apply Know the explicit form of given. Know the derivatio	the element	stiffness matrix and	the equivalent not	dal force vector are		

	<ul> <li>triangular shape and different orders of approximation. Be able to understand that the resulting expressions for the shape functions are applicable to axisymmetric solid elements, as well as for many plate and shell elements. Know the formulation of 2D solid elements of arbitrary shape (i.e. irregular quadrilateral and triangular elements with straight or curved sides) using the isoparametric formulation and numerical integration. Know and be able to understand the essential ideas of the organization of a general FEM computer program applicable to elements of different shape and approximation order.</li> <li>Topic 5: Be able to notice of the use is made of the concepts explained in the previous two chapters, such as the derivation of the element shape functions, the isoparametric formulation and numerical integration. Discuss several applications of axisymmetric solids and structures.</li> </ul>
	Topic 6: Know and be able to understand the formulation 3D solid elements of tetrahedral and hexahedral shapes. Be able to notice that 3D solid elements allow the FEM analysis of any structure. Know the details of the derivation of the stiffness matrix and the equivalent nodal force vector for the simple 4-noded tetrahedral element. Know and be able to apply the formulation of higher order 3D solid elements is explained using the isoparametric formulation and numerical integration. Discuss several applications of 3D solid elements to a wide range of structures such as dams, buildings, historical constructions and mechanical parts.
	Topic 7: Know and be able to understand several topics of general interest for FEM analysis. These include the treatment of inclined supports, the blending of elements of different types, the study of structures on elastic foundations, the use of substructuring techniques, the procedures for applying constraints on the nodal displacements, the computation of stresses at the nodes and the key concepts of error estimation and adaptive mesh refinement strategies.
Cross	1. Acquire a basic vocabulary while specific field of engineering.
competences:	<ol> <li>Capacity for effective and correct oral or written, on issues within the field of engineering.</li> </ol>
	3. Ability to read, correctly interpret and understand texts, figures and tables in scientific and technical literature.
	4. Acquire independent learning skills in different fields of engineering.
	5. Development and improvement of the ability to organize and plan of study and personal work-
	6. Development and improvement of behaviors, skills and ways to study and work cooperatively.
	7. Ability to develop a self-learning evaluation.
	8. Developing a professional attitude proactive, assertive and communicative.
	9. Highlight the need to retain an appropriate level in algebra and mathematical calculation as inevitable tools in engineering practice.

# CONTENTS

## TOPIC 1: INTRODUCTION TO THE FINITE ELEMENT METHOD FOR STRUCTURAL ANALYSIS (4T+4P)

What is the Finite Element Method? Analytical and numerical methods. What is a finite element? Structural modelling and fem analysis. Discrete systems. Bar structures. Direct assembly of the global stiffness matrix. Derivation of the matrix equilibrium equations for the bar using the principle of virtual work. Derivation of the bar equilibrium equations via the minimum total potential energy principle. Plane frameworks. Treatment of prescribed displacements and computation of reactions. Introduction to the finite element method for structural analysis. The value of finite element computations for structural design and verification.

## **TOPIC 2: FINITE ELEMENTS FOR AXIALLY LOADED RODS** (4T+4P)

Introduction. Axially loaded rod. Axially loaded rod of constant cross section. Discretization in one linear rod element. Derivation of the discretized equations from the global displacement interpolation field. Axially loaded rod of constant cross section. Discretization in two linear rod elements. Generalization of the solution with n linear rod elements. Extrapolation of the solution from two different meshes. Matrix formulation of the element equations. Summary of the steps for the analysis of a structure using the fem.

**TOPIC 3: ADVANCED ROD ELEMENTS AND REQUIREMENTS FOR THE NUMERICAL SOLUTION** (3T+4P) Introduction. One dimensional c0 elements. Lagrange elements. Isoparametric formulation and numerical integration. Numerical integration. Steps for the computation of matrices and vectors for an isoparametric rod element. Basic organization of a finite element program. Selection of element type. Requirements for convergence of the solution. Assessment of convergence requirements. Other requirements for the finite element approximation the patch test. Some remarks on the compatibility and equilibrium of the solution. Convergence requirements for isoparametric elements. Error types in the finite element solution. Concluding remarks.

## TOPIC 4: BIDIMENSIONAL SOLIDS (8T+8P)

Introduction. Two dimensional elasticity theories. Finite element formulation. Three-noded triangular element. The four noded rectangular element. Performance of the 3-noded triangle and the 4-noded rectangle. Derivation of the shape functions for co two dimensional elements. Lagrange rectangular elements. Serendipity rectangular elements. Shape functions for c0 continuous triangular elements. Analytic computation of integrals over Rectangles and straight-sided triangles. General performance of triangular elements. Enhancement of 2d elasticity elements using drilling rotations. Isoparametric quadrilateral elements. Isoparametric triangular elements. Numerical integration in two dimensions. Numerical integration of the element matrices and vectors. Computer programming of  $\mathbf{K}_{(e)}$  and  $\mathbf{f}_{(e)}$ . Optimal points for computing strains and stresses. Selection of the quadrature order. Performance of 2d isoparametric solid elements. The patch test for solid elements. Applications. Concluding remarks.

### TOPIC 5: AXISYMMETRIC SOLIDS (2T+2P)

Introduction. Finite element formulation. Three-noded axisymmetric triangle. Other rectangular or straight-sided triangular axisymmetric solid elements. Isoparametric axisymmetric solid elements. Analogies between the finite element formulations for plane elasticity and axisymmetric solids. Examples of application. Concluding remarks.

### **TOPIC 6: THREE DIMENSIONAL SOLIDS** (3T+4P)

Introduction. Basic theory. Finite element formulation. The four-

Noded tetrahedron. Other 3d solid elements. Right prisms. Straight-edged tetrahedra. Computation of element integrals. 3d isoparametric elements. Numerical integration. Numerical integration of element matrices. Performance of 3d solid elements. Examples. Concluding remarks.

### TOPIC 7: MISCELLANEOUS (3T+2P)

Introduction. Boundary conditions in inclined supports. Joining dissimilar elements. Displacement constraints. Nodal condensation and substructures. Structural symmetry. Structures on elastic foundation. Computation of nodal stresses. Error estimation and mesh adaptivity.

#### Laboratory:

- 1. Generating geometries in the GiD program. Part 1
- 2. Generating geometries in the GiD program. Part 2
- 3. Generating meshes in the GiD program. Part 2.
- 4. Generating meshes in the GiD program. Part 2.
- 5. 2D Solids examples. Part 1.
- 6. 2D Solids examples. Part 2.
- 7. 2D Solids examples. Part 3.
- 8. Axisymmetric Solids examples. Part 1.
- 9. Axisymmetric Solids examples. Part 2.
- **10**. 3D Solids examples. Part 1.
- 11. 3D Solids examples. Part 2.
- 12. 3D Solids examples. Part 3.
- 13. Working in the final project. Part 1
- 14. Working in the final project. Part 2

Out of the classroom activities:

Practical problem solving using technical codes, technical documentation and software of FEM.

## Students weekly work expressed in hours:

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1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Total
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2	2	2	2	2	2	2	2	2	2	2	2	2	2	0						28
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3	3	3	3	3	3	3	3	3	3	3	3	3	3	3						45
3	3	3	3	3	3	3	3	3	3	3	3	3	3	3						45
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10	10	10	10	10	10	10	10	10	10	10	10	10	10	10						150
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     2       2       2       2       2       2       2       2       0       11       12       13       14       15       16       17       18         2       2       2       2       2       2       2       2       2       2       0       16       17       18         3       3       3       3       3       3       3       3       3       3       3       3       3       3       3       16       17       18         3       3       3       3       3       3       3       3       3       3       3&lt;</td><td>1       2       3       4       5       6       7       8       9       10       11       12       13       14       15       16       17       18       19         2       2       2       2       2       2       2       2       2       2       2       2       2       2       2       2       2       0       11       12       13       14       15       16       17       18       19         2       2       2       2       2       2       2       2       2       2       2       0         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## **Teaching/Learning method:**

The course uses the narrative method by 50%, individual work 25%, and project-based learning by 25%.

### Main bibliographic resources:

1. OÑATE, EUGENIO "Structrural analysis with finite elements methods. Linears Static. Vol. 1 Basic and Solids". Springer. Barcelona, 2009.

### **Complementary bibliographic resources:**

1. BATHE, KLAUS-JÜRGEN "Finite Elements Procedures". Prentice-Hall Inc, Upper Saddle River, New Jersey 07458, 1996.

Assessment and qu	sessment and qualification:											
Midterm tests:	30 %	Exercises/Problems:	0 %	Final exam:	35 %							
Out of classroom:	0%	Practice:	35%	Group work:	0 %							
The evaluation was	conducted	through the assessment	by the professor.									